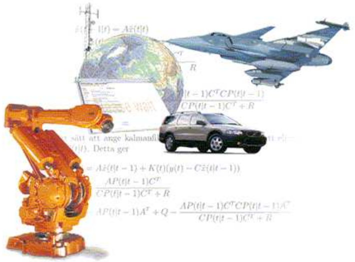
Licentiate's Presentation

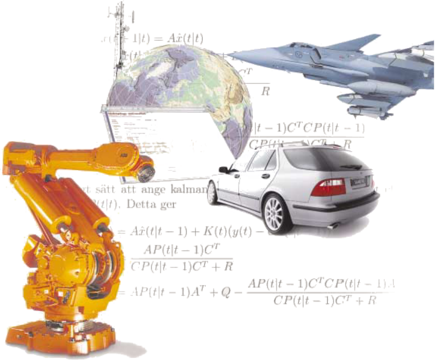
Fundamental Estimation and Detection Limits in Linear Non-Gaussian Systems



Gustaf Hendeby

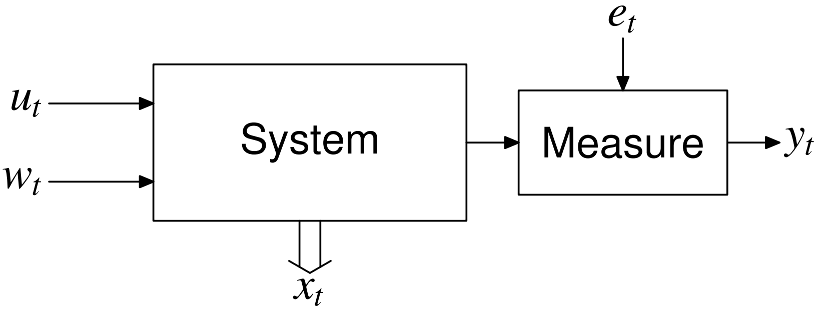
Automatic Control
Department of Electrical Engineering
Linköpings universitet

Motivation



- Estimation and detection are used everywhere
- Vital functions rely on it
- Information is expensive

System description



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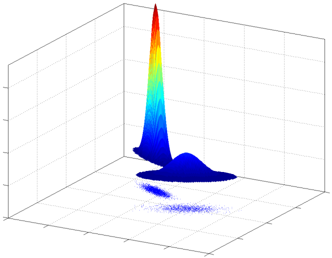
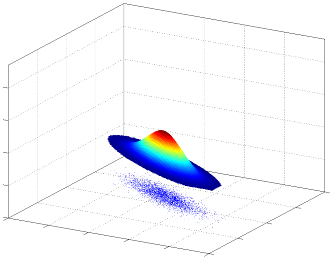
Noise approximation

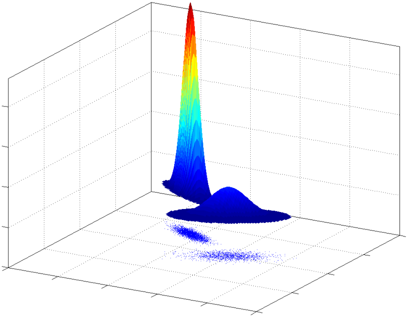


True distribution



Gaussian approximation

Noise approximation



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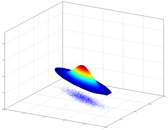

Gaussian approximation

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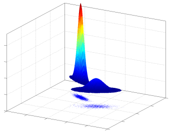

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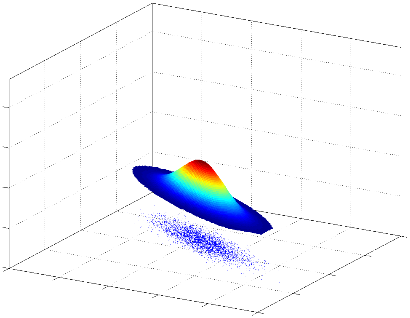

Gaussian approximation

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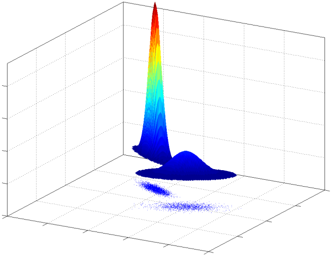

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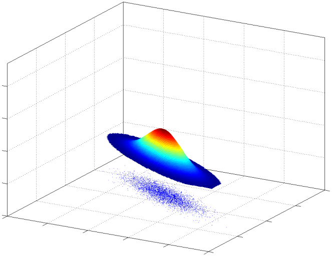

Gaussian approximation

Outline

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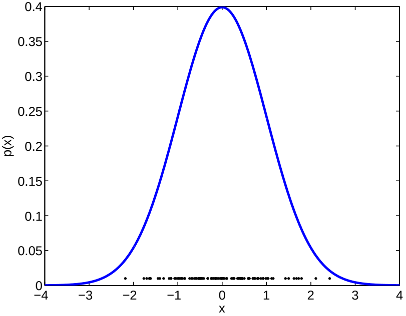
Outline

Noise

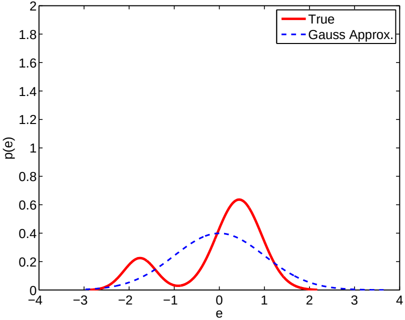
- Random effects
  - Measurement noise
  - Process noise
- More or less informative
- Description:
  - PDF $p(x)$
  - Expected value: $\mathrm{E}(x) = \mu$
  - Variance: $\mathrm{var}(x) = \Sigma$



$$p(x) = \mathcal{N}(x; 0, 1)$$

Information and Accuracy

$$y_t = \theta + e_t$$



e_t

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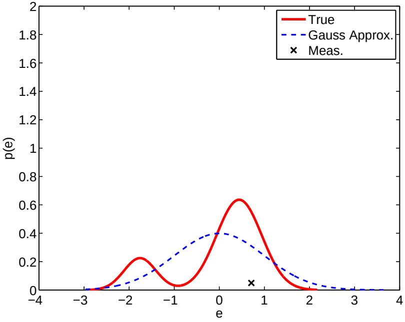

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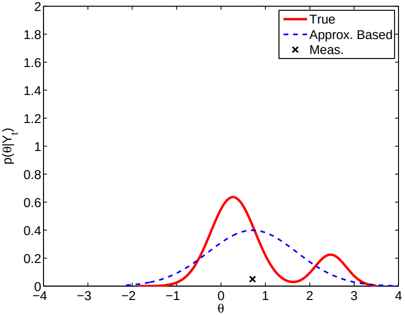

$\theta | \mathbb{Y}_t$

Information and Accuracy

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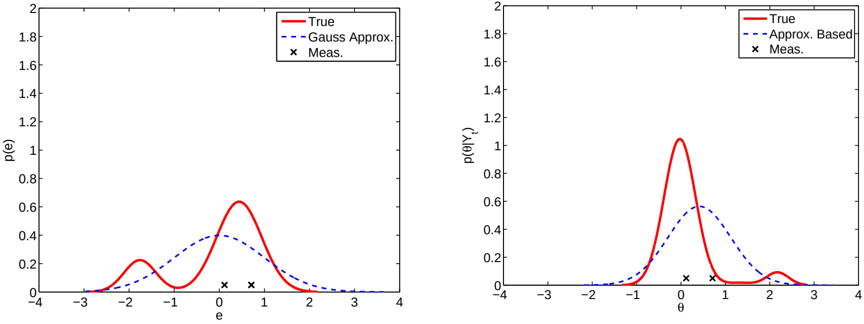

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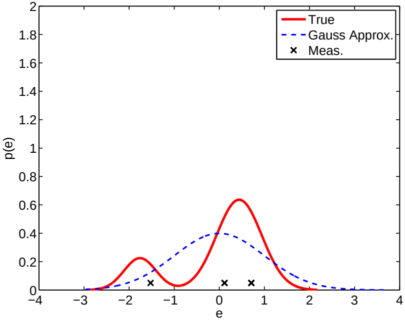

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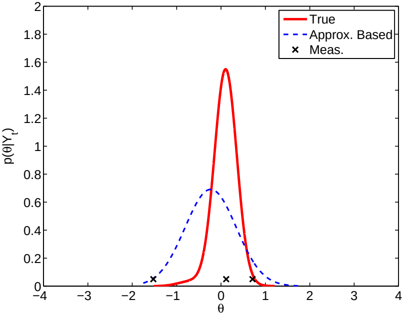

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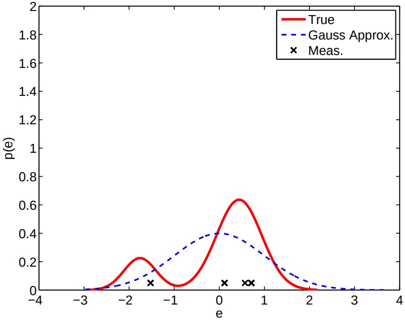
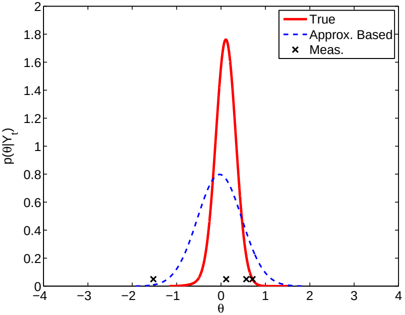


e_t



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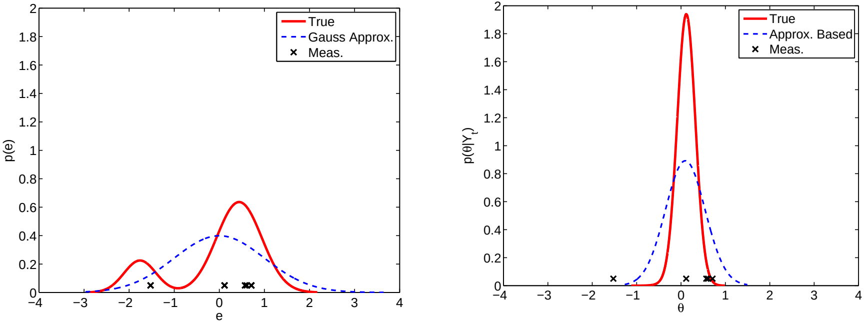

e_t $\qquad$ $\theta | \mathbb{Y}_t$

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Definitions: information measures

- Fisher information (true parameter θ^0):

$$\mathscr{I}_x(\theta) = -\operatorname{E}_x\left(\Delta_\theta^\theta \log p(x|\theta)\,\Big|_{\theta=\theta^0}\right)$$

- Intrinsic accuracy (true mean μ^0):

$$\mathscr{I}_x = -\operatorname{E}_x\left(\Delta_x^x \log p(x|\mu^0)\right)$$

- Relative accuracy:

$$\Psi_x = \operatorname{var}(x)\,\mathscr{I}_x$$

- Kullback-Leibler information:

$$\mathscr{I}^{\mathrm{KL}}\big(p(\cdot), q(\cdot)\big) = \int p(x)\log\left(\frac{p(x)}{q(x)}\right)dx$$

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Example: intrinsic accuracy

Assume

$$y_i = \theta + e_i, \quad e_i \sim \mathcal{N}(\mu = 0, \Sigma),$$

then the intrinsic accuracy is

$$\begin{aligned}
\mathcal{I}_e &= -\operatorname{E}_e\Big(\Delta_e^e \log \mathcal{N}(e;\mu,\Sigma)\Big) \\
&= -\operatorname{E}_e\Big(\Delta_e^e \log \frac{1}{\sqrt{2\pi\Sigma}} e^{-\frac{(e-\mu)^2}{2\Sigma}}\Big) \\
&= \operatorname{E}_e\Big(\Delta_e^e \big(\log\sqrt{2\pi\Sigma} + \frac{(e-\mu)^2}{2\Sigma}\big)\Big) \\
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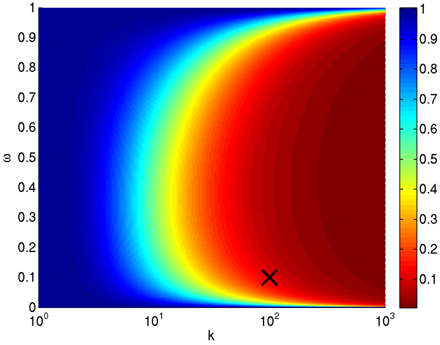
\end{aligned}$$

Intrinsic Accuracy: outliers

$$p_1(x;\omega,k) = (1-\omega)\mathcal{N}(x;0,\Sigma) + \omega\mathcal{N}(x;0,k\Sigma)$$



- Inverse relative accuracy: $\Psi_x^{-1} = \big(\operatorname{cov}(x)\mathscr{I}_x\big)^{-1}$
- $\Sigma^{-1} := 1 + (k-1)\omega$, to get $\operatorname{cov}(x) = 1$
- Red is informative, blue is not
- $k = 1$ yields Gaussian distribution

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Outline

Estimation

- Extract hidden information
- Find $p(x_t|\mathbb{Y}_\tau)$:
    - Measurement update phase:

$$p(x_t|\mathbb{Y}_\tau) = \frac{p(y_\tau|x_t)p(x_t|\mathbb{Y}_{\tau-1})}{p(y_\tau|\mathbb{Y}_{\tau-1})}$$

    - Time update phase:

$$p(x_{t+1}|\mathbb{Y}_\tau) = \int p(x_{t+1}|x_t)p(x_t|\mathbb{Y}_\tau)\, dx_t,$$

- Approximation of $p(x_t|\mathbb{Y}_\tau)$ often needed

State-Space Model

- General State-Space Model:

$$\begin{aligned} x_{t+1} &= f(x_t, w_t) \\ y_t &= h(x_t, e_t) \end{aligned}$$

- Linear State-Space Model:

$$\begin{aligned} x_{t+1} &= F_t x_t + G_t w_t \\ y_t &= H_t x_t + e_t \end{aligned}$$

- $Q_t = \text{cov}(w_t)$ and $R_t = \text{cov}(e_t)$

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Estimation: algoritms

Kalman Filter

The best linear unbiased estimator (BLUE)

1. Initiate: $\hat{x}_{0|-1}$, $P_{0|-1}$
2. Measurement update phase:

$$K_t = P_{t|t-1} H_t^T \left(H_t P_{t|t-1} H_t^T + R_t \right)^{-1}$$
$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t \left(y_t - H_t \hat{x}_{t|t-1} \right)$$
$$P_{t|t} = \left(I - K_t H_t \right) P_{t|t-1}$$

3. Time update phase:

$$\hat{x}_{t+1|t} = F_t \hat{x}_{t|t}$$
$$P_{t+1|t} = F_t P_{t|t} F_t^T + Q_t$$

- EKF, IEKF, UKF, filter banks

Particle Filter

Asymptotically (in N) correct PDF

1. Initiate: $\{x_0^{(i)}\}_{i=1}^N \sim p(x_0)$, $\{\omega_{0|-1}^{(i)}\}_{i=1}^N = \frac{1}{N}$
2. Measurement update phase:

$$\omega_{t|t}^{(i)} = \frac{p(y_t|x_t^{(i)})\,\omega_t^{(i)}}{\sum_j p(y_t|x_t^{(j)})\,\omega_t^{(j)}}$$

3. Resample!
4. Time update phase:
   $\{x_{t+1}^{(i)}\}_{i=1}^N \sim q(x_{t+1}|x_t^{(i)}, \mathbb{Y}_t)$,

$$\omega_{t+1|t}^{(i)} = \frac{\omega_{t|t}^{(i)}\, p(x_{t+1}^{(i)}|x_t^{(i)})}{q(x_{t+1}^{(i)}|x_t^{(i)}, \mathbb{Y}_t)}$$

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Parametric Cramér-Rao Lower Bound (CRLB)

- Estimation performance, assuming correct trajectory exists.
- Bound given by:

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1} H_t P_{t|t-1}$$
$$P_{t+1|t} = F_t P_{t|t} F_t^T + G_t Q_t G_t^T,$$

initialized with $P_{0|-1}^{-1} = \mathscr{I}_{x_0}$ and with

$$F_t^T = \nabla_{x_t} f(x_t, w_t^0)\big|_{x_t = x_t^0}, \qquad G_t^T = \nabla_{w_t} f(x_t^0, w_t)\big|_{w_t = w_t^0},$$
$$H_t^T R_t^{-1} H_t = -\operatorname{E}_{y_t}\left(\Delta_{x_t}^{x_t} p(y_t|x_t)\right), \quad Q_t^{-1} = -\operatorname{E}_{x_t}\left(\Delta_{w_t}^{w_t} p(x_t|w_t^0)\right).$$

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Posterior Cramér-Rao Lower Bound (CRLB)

- Estimation performance, assuming only trajectory distribution.
- Bound given by:

$$P^{-1}_{t+1|t} = Q^{-1}_t - S^T_t(P^{-1}_{t|t-1} + R^{-1}_t + V_t)^{-1}S_t$$
$$P^{-1}_{t+1|t+1} = Q^{-1}_t + R^{-1}_{t+1} - S^T_t(P^{-1}_{t|t} + V_t)^{-1}S_t$$

initiated with $P^{-1}_{0|-1} = \mathscr{I}^{-1}_{x_0}$, $P^{-1}_{0|0} = (P^{-1}_{0|-1} + R^{-1}_0)^{-1}$, with:

$$V_t = -\operatorname{E}_{x_t,w_t}\left(\Delta^{x_t}_{x_t}\log p(x_{t+1}|x_t)\right), \qquad R^{-1}_t = -\operatorname{E}_{x_t,y_t}\left(\Delta^{x_t}_{x_t}\log p(y_t|x_t)\right),$$
$$S_t = -\operatorname{E}_{x_t,w_t}\left(\Delta^{x_{t+1}}_{x_t}\log p(x_{t+1}|x_t)\right), \quad Q^{-1}_t = -\operatorname{E}_{x_t,w_t}\left(\Delta^{x_{t+1}}_{x_{t+1}}\log p(x_{t+1}|x_t)\right).$$

Linear Systems CRLB

- Parametric and posterior CRLB are identical:

$$P_{t|t} = \left(P_{t|t-1}^{-1} + H_t^T \mathscr{I}_{e_t} H_t\right)^{-1},$$
$$P_{t+1|t} = F_t P_{t|t} F_t^T + G_t \mathscr{I}_{w_t}^{-1} G_t^T,$$

  initiated with $P_{0|-1} = \mathscr{I}_{x_0}^{-1}$
- Effects of $\mathscr{I}_{w_t}$ and $\mathscr{I}_{e_t}$ examined
- Compare to BLUE performance given by Kalman filter (same expression)

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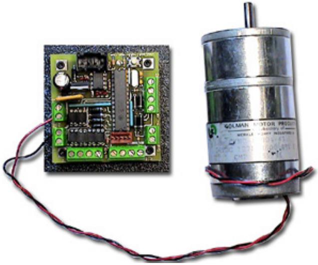
Example: DC motor setup

State-space model:

$$x_{t+1} = \begin{pmatrix} 1 & 1-e^{-1} \\ 0 & e^{-1} \end{pmatrix} x_t + \begin{pmatrix} e^{-1} \\ 1-e^{-1} \end{pmatrix} w_t$$

$$y_t = \begin{pmatrix} 1 & 0 \end{pmatrix} x_t + e_t$$



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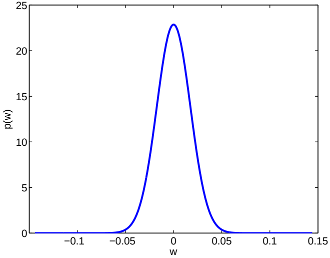
$$y_t = \begin{pmatrix} 1 & 0 \end{pmatrix} x_t + e_t$$

with the noise

$$w_t \sim \mathcal{N}\left(0, \left(\tfrac{\pi}{180}\right)^2\right)$$

$$e_t \sim 0.9\mathcal{N}\left(0, \left(\tfrac{\pi}{180}\right)^2\right) + 0.1\mathcal{N}\left(0, \left(\tfrac{10\pi}{180}\right)^2\right)$$

w_t



$$\mathrm{var}(w_t) = 3.0 \cdot 10^{-4}$$

$$\mathcal{I}_{w_t} = 3.3 \cdot 10^{3}$$

$$\Psi_{w_t} = 1$$

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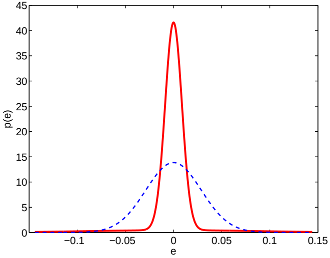
with the noise

$$w_t \sim \mathcal{N}\left(0, \left(\tfrac{\pi}{180}\right)^2\right)$$

$$e_t \sim 0.9\mathcal{N}\left(0, \left(\tfrac{\pi}{180}\right)^2\right) + 0.1\mathcal{N}\left(0, \left(\tfrac{10\pi}{180}\right)^2\right)$$

e_t



$$\operatorname{var}(e_t) = 8.3 \cdot 10^{-4}$$

$$\mathcal{I}_{e_t} = 1.1 \cdot 10^{4}$$

$$\Psi_{e_t} = 9.0$$

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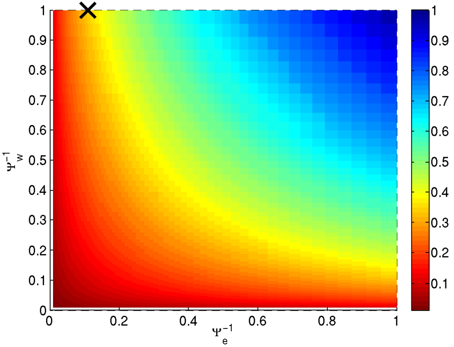
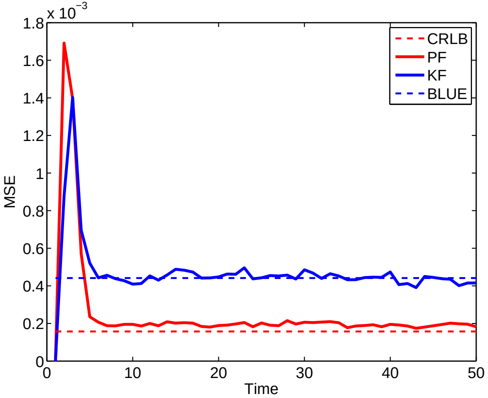
Example: filtering analysis



- Normalized filtering performance
- Red is improvement, blue is not
- Note axis

Example: filtering result



- Monte Carlo simulations
- Dashed lines indicate asymptotic limits
- Note improvement

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Observations

- Improved performance with nonlinear filter on non-Gaussian noise.
- Comparing CRLB and BLUE performance indicates the gain.
- System properties and used methods affect actual gain.

Outline

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Detection

- Determine if a change/fault has occurred
- Decide between hypotheses; $\mathscr{H}_0$ and $\mathscr{H}_1$
- Common design criteria:
  - Minimize probability of false alarm

$$P_{\text{FA}} = \Pr(\text{decide } \mathscr{H}_1 | \mathscr{H}_0)$$

  - Maximize probability of detection

$$P_{\text{D}} = \Pr(\text{decide } \mathscr{H}_1 | \mathscr{H}_1)$$

Detection: methods

- Decide using

$$L(\mathbb{Y}) \underset{\mathscr{H}_0}{\overset{\mathscr{H}_1}{\gtrless}} \gamma$$

- Generalized likelihood ratio (GLR) test statistic

$$L(\mathbb{Y}) = \frac{\sup_{f|\mathscr{H}_1} p(\mathbb{Y}|\mathscr{H}_1)}{\sup_{f|\mathscr{H}_0} p(\mathbb{Y}|\mathscr{H}_0)}$$

- Composite hypotheses are more difficult and more common

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Detection Models

- Residuals:
$$r_t = y_t - h(x_t, f_t^0)$$
- Fault models:
$$f_t = \varphi_t^T \theta$$
- Stacked linear residuals $\mathbb{R}_t = \begin{pmatrix} r_{t-L+1}^T & r_{t-L+2}^T & \dots & r_t^T \end{pmatrix}^T$:
$$\mathbb{R}_t = \mathbb{Y}_t - \mathscr{O}_t x_{t-L+1} = \bar{H}_t^\theta \theta + \bar{H}_t^v \mathbb{V}_t$$
- Known initial state and detection in parity-space can be described this way

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GLR Performance

- Asymptotically (in information) uniformly most powerful (UMP)
- Test statistics

$$L'(\mathbb{Y}) := 2\log L(\mathbb{Y}) \overset{a}{\sim} \begin{cases} \chi^2_{n_\theta}, & \text{under } \mathscr{H}_0 \\ \chi'^2_{n_\theta}(\lambda), & \text{under } \mathscr{H}_1 \end{cases},$$

with

$$\begin{aligned} \lambda &= \theta^{1T} \bar{H}_t^{\theta T} \mathscr{I}_{\bar{H}_t^v \mathbb{V}} \bar{H}^\theta \theta^1 \\ &= \theta^{1T} \bar{H}_t^{\theta T} (\bar{H}_t^v \mathscr{I}_{\mathbb{V}}^{-1} \bar{H}_t^{vT})^{-1} \bar{H}^\theta \theta^1 \end{aligned}$$

Example: DC motor setup

State-space model:

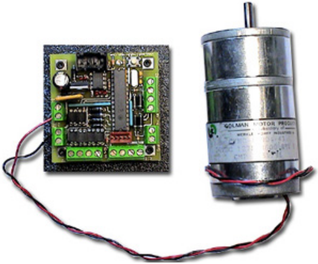
$$x_{t+1} = \begin{pmatrix} 1 & 1-e^{-1} \\ 0 & e^{-1} \end{pmatrix} x_t + \begin{pmatrix} e^{-1} \\ 1-e^{-1} \end{pmatrix} (w_t + f_t)$$

$$y_t = \begin{pmatrix} 1 & 0 \end{pmatrix} x_t + e_t$$

with the noise

$$w_t \sim \mathcal{N}\left(0, \left(\tfrac{\pi}{180}\right)^2\right)$$

$$e_t \sim 0.9\mathcal{N}\left(0, \left(\tfrac{\pi}{180}\right)^2\right) + 0.1\mathcal{N}\left(0, \left(\tfrac{10\pi}{180}\right)^2\right)$$



Example: DC motor setup

State-space model:

$$x_{t+1} = \begin{pmatrix} 1 & 1-e^{-1} \\ 0 & e^{-1} \end{pmatrix} x_t + \begin{pmatrix} e^{-1} \\ 1-e^{-1} \end{pmatrix} (w_t + f_t)$$

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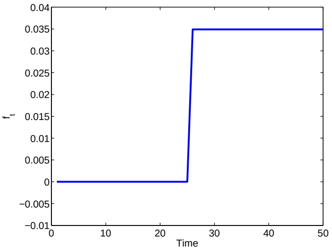
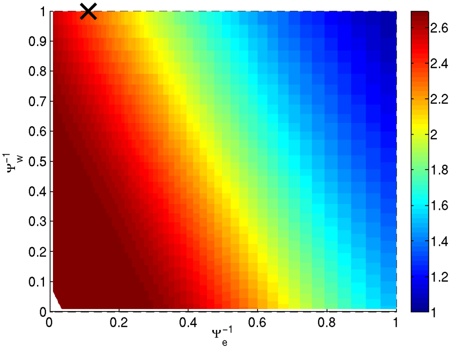


$$f_t = \begin{cases} 0, & t \leq 25 \\ \frac{2\pi}{180}, & t > 25 \end{cases}$$

Example: detection analysis



- Normalized detection performance, $P_{FA} = 1\%$
- Red is improvement, blue is not
- Note axis
- Level 1 (blue) equals $P_D = 37\%$

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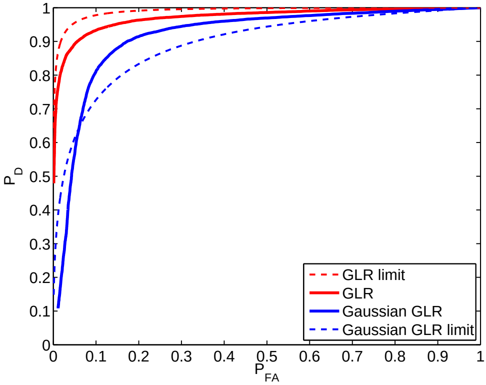
Example: detection result



- Monte Carlo simulations
- Dashed lines indicate asymptotic limits
- Note improvement

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Observations

- Utilizing non-Gaussian effects may substantially improve detection performance.
- It is sometimes difficult to predict the effect of Gaussian approximations.
- The system determines how difficult it is to improve performance.

Outline

1. Introduction
2. Noise and Information
3. Estimation Limits
4. Detection Limits
5. Conclusions

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Conclusions

- Consider non-Gaussian effects
- Estimation: compare CRLB to BLUE performance
- Detection: compare asymptotic GLR performance under different assumptions
- Improvement shown in simulations

- Further work
  - When are asymptotic results reached?
  - Other performance measures?
  - Robustness?
  - Treat nonlinear systems

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